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Analysis of Viscoelasticity in Pipe Systems in Frequency Domain

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Abstract

Unsteady flow is the result of turbulence (valve shut, pump deactivation, etc.) in the steady flow. A sudden change in the velocity of fluid, flowing inside the tube, brings about an abrupt change in pressure and water hammer occurs as the result. Because of internal fluid-structure interaction inside the pipe, this phenomenon effects on the pipe system. During water hammer phenomenon, considerable dynamic forces are exerted on the pipe. If these forces cause deformation of the structure, Fluid-structure interaction occurs. Thus, the present research intended to investigate the pip dynamic behaviour in frequency domain. In time models proposed for solving water hammer equations in pipe structure, because of using interpolation a great deal of time was required for transmission of hydraulic parameters to structure equations and vice versa. Therefore, by simplicity of water hammer differential equation in frequency domain, the problem was solved in this domain. The comparison made between the achieved results of this research and those obtained by other researchers confirmed the accuracy and efficiency of this method.

Key words: Unsteady Flow, Fluid-Structure interaction, Water Hammer, Frequency Domain

1- Introduction

During water hammer, considerable dynamic forces are exerted on the structure. Fluid-structure interaction occurs where these forces cause deformation on the structure. Therefore, it is impossible to investigate pipe behaviour or fluid separately so a simultaneous investigation needs to be undertaken- In other words, we have to study interaction mechanism. In some materials, molecules' array change gradually toward each other as a result of external loading. It causes deformation in addition to deformation occurred at the beginning of loading. Viscoelastic materials have both short and long-time deformation as a result of loading [1]. So far, a variety of models have been developed for describing these substances mathematically [2]. These models with their specific array of some springs and damper can be used to describe viscoelastic substances behaviour [10]. Here, we apply Kelvin-Woigt model for describing viscoelastic substances behavior (Fig. 1). In viscoelastic problems, Matching Principle is used, when frequency solution is under consideration. Then, deferential equation is written in Laplace domain, and thus, the exact solution can be achieved in Laplace domain [4]. The idea of fluid-structure interaction in pipes, first developed by Skalac in 1956 with presenting interaction equations concerning water hammer [5]. Afterward, it was investigated uninterruptedly and a wide variety of procedures including Coupling, semi coupling, and various algorithms recommended for numerical modeling among which we can refer to solving interaction equation of pipe vibration through method of characteristic lines proposed by wiggert and Taijsseling, solving structure equations through finite element, solving hydraulic equations through method of characteristic lines and solving fluid-structure interaction in pipes analytically [6].

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2- Classifications of Various Types of Coupling Modelling

Coupling procedures which are used for solving problems of pipes fluid-structure interaction (FSI) can be classified as below, according to differential equations used in each procedure:

1. Model of two differential Equations: In this model, only two hydraulic differential equations can be solved (continuity and momentum) and thus, the calculated pressure and velocity are used for analysing structure equations. In this method which is in reality a semi coupling method, the values calculated by solving hydraulic equations are considered as an external uploading in structure equations. This analytical method is also known as classical water hammer solution [5].
2. Model of four first-order differential equations: these four equations involve two continuity and momentum equations that are hydraulic equations and the structure axial vibration equation shifts into two first-order differential equations because it is a second-order equation. This model can be used in direct pipes with axial movements. In addition to pressure and velocity, axial stress and axial velocity of pipe wall are other variables that can be seen in these four first-order differential equations [5].
3. Model of six first-order differential equations: this model is required only if radial inertia forces are needed. In this model, hoop stress and radial velocity are unknown in addition to what mentioned as unknown

parts in the former model [5].

4. Model of fourteen first-order differential equations: these fourteen first-order equations are: two hydraulic equations, one axial vibration equation that changes into two first-order differential equation as it is a second-order equation, one rotational vibration equation that again is a second-order equation and therefore changes into two first-order differential equations, and two bending vibration equations in two screens of xy and xz that can come to eight first-order differential equations for the fact that bending vibration equations are fourth – order. This method can model pipe and fluid axial vibration on vibration screen and out of vibration screen and even rotational vibration in 3-D position in systems [5].

3- Various types of coupling mechanism

Three major mechanisms have been discovered by investigation into FSI phenomenon.

1. Poisson coupling mechanism: Poisson coupling mechanism is the result of Poisson ratio in pipe materials that change radial stress into axial stress. It can be investigated by using an expression that is dependent on Poisson ratio of materials and exists in hydraulic equations, as well as, structure. This effect can cause sudden changes in fluid pressure diagrams and structure stress [3].

2. Friction coupling mechanism: rises from fluid friction against pipe wall that reduces pressure and pipe stress (damping). For this reason, a large number of researchers ignore this effect which is just for certainty [3].

3. Junction coupling mechanism: it occurs when some junction of the pipe structure are not perfectly connected on the ground. In some cases, it is observed that the effect of this coupling is more significant than Poisson coupling and thus it can intensify stress and consequently destroy the structure [3].

4- Mechanical behaviour of viscoelastic Materials

Viscoelastic materials exhibit both fluids and solids characteristics. To model mechanical behaviour of a linear elastic solid substance, a spring is usually used and in one-dimensional position it is modelled as FS= K_1U in which F is force, U stands for movement, and S represents spring. To model mechanical behavior of a linear viscous fluid, a linear viscous damper is usually used and in one-dimensional position it is modeled as FD=$K_2\dot{U}$ in which D represents damper, and spot over U indicates displacement derivative with respect to time [7].

One way to get the answer of a linear viscoelastic substance is to consider a system containing a spring which is located parallel with a damper. In this case, it is obvious that the entire force is F=F_s+F_D, and therefore F=K_1U=$K_2\dot{U}$. In this model, the relationship between stress σ and strain ε (set of spring and damper) is defined as below:

$V=\frac{1}{\pi R^2}\int_0^R 2\pi r U_Z\, d_r$

$$p_0\sigma = q_0\,\epsilon + q_1\dot{\epsilon} \quad p_0 = 1\,, q_0 = E\,, q_1 = \mu \qquad (1)$$

In which, E is spring modulus elasticity, μ is damper viscosity, ε indicates strain rate change. It can be proved that the following relationship exists in the generalized model of Kelvin- Woigt

$$p_0\sigma + \sum_{k=1}^{N_{K_V}} \rho_K \frac{d\sigma^k}{dt^k} = q_0\varepsilon + \sum_{k=1}^{N_{K_V}} q_k \frac{d^k\varepsilon}{dt^k} \qquad (2)$$

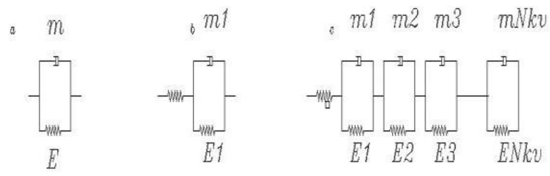

Figure 1: mechanical presentation of a viscoelastic solid substance (a) Kelvin- Woigt (b) three-parameter model of Kelvin- Woigt (c) generalized model of Kelvin- Woigt.

5- The Equations

For linear viscoelastic substances in frequency domain, the relationship between stress and strain is [4]:

$$\overline{\varepsilon(s)} = \bar{\sigma}(S).s\ \overline{J(S)} \qquad (3)$$

In the above formula, according to generalized model of Kelvin- Woigt, creep compliance function is computed $\overline{J(S)}$ as below:

$$\overline{J(S)} = (\frac{1}{S}\,.\ \sum_{K=1}^{N_{K_V}} J_K\ - \sum_{K=1}^{N_{K_V}} \frac{J_K\,\tau_K}{S\,\tau_K+1}\ ) \qquad (4)$$

In which $J_0=1/E$ represents an immediate reaction of viscoelastic materials, $J_k=1/E_k$ indicates spring creep compliance of K Kelvin Woigt, and E_K shows elasticated modulus K spring, τ_K is the delaying time of K damper. μ_K in $\tau_k=\mu_k/E_k$ is K viscoelastic damper.

5.1 Continuity and momentum equations

To obtain above equation in a pipe system, Navier-Stokes is written in two-dimensional r-z cylindrical coordinate system [2]. These equations involve a continuity equation and two momentum equations in radial and axial direction with variables of axial velocity V_Z, radial velocity V_r , fluid pressure P, and fluid density ρ_f, moreover, it relates Equation of state, pressure, and fluid density. Continuity Equation:

$$\frac{\partial v}{\partial z} + \frac{1}{K}\frac{\partial p}{\partial t} + \frac{2}{R}\dot{u}_r\,\big|_{r=R} = 0,\ \ \dot{u}_r\,\big|_{r=R} = V_r\,\big|_{r=R} \qquad (5)$$

And momentum equation in axial direction:

$$\frac{\partial v}{\partial t} + \frac{1}{\rho_f}\frac{\partial p}{\partial z} = -\frac{2}{\rho_f R}\tau_0 + g\sin\theta \qquad (6)$$

In which velocity (v) and pressure (p) are:

$$V=\frac{1}{\pi R^2}\int_0^R 2\pi r U_Z\, d_r \qquad (7)$$

$$V=\frac{1}{\pi R^2}\int_0^R 2\pi r \rho\, d_r \qquad (8)$$

In which V_Z and P are distribution function and flow pressure respectively. In these expressions, Z represents pipe axis length, t is time, g stands for gravity, R indicates pipe internal radius, and ρ_f is fluid density.

Assuming that the pipe wall is thin and according to small strain theory, the following expression is computed for viscoelastic pipes in continuity differential equation:
Continuity equation:

$$\frac{\partial V}{\partial z}+\frac{1}{K}\frac{\partial P}{\partial t}-2\upsilon\frac{\partial \dot{u}_z}{\partial z}=(\boldsymbol{\upsilon}^2-1)\rho_f g\frac{D}{e}\frac{\partial I_{\tilde{H}}}{\partial t} \quad (9)$$

In this expression, $\mathring{u}_Z$ is pipe axial velocity, f shows Darcy-Weisbach friction coefficient , D represents pipe internal diagonal, υ is Poisson ratio, ρ_f is fluid density, e shows pipe internal thickness, $I_{\tilde{H}}$ is delaying environmental strain coefficient, and C_f is pressure wave velocity. We have:

$$C_f=(\rho_f\left(\frac{1}{K}+(1-\nu^2)\frac{D}{Ee}\right)^{-\frac{1}{2}} \quad (10)$$

$$I_{\bar{H}}=\int_0^t \bar{H}\,(t-s).\frac{dJ}{ds}(S)ds \quad (11)$$

Momentum equation:

$$\frac{\partial \text{v}}{\partial \text{t}}+g\frac{\partial H}{\partial z}=\frac{-fV|V|}{2D} \quad (12)$$

5.2 Axial vibration equations

Due to the fact that Poisson coupling exists in continuum equation, and therefore there is a need for computing axial displacement on various pipe spots, axial vibration equations again need to be extracted. Axial vibration, itself, is under the influence of pipe wall viscoelasticity. The first step in modeling axial vibration is writing motion equation on axial direction Z and radial direction r in a two-dimensional state on a cylindrical coordinate system. These two equations are considered as momentum balance expression on axial and radial directions. The effects of Bending stiffness, rotational inertia, and transverse shear deformation are disregarded. These hypotheses are known as hypotheses of waves with long wave length [5].

$$\frac{\partial \dot{u}_z}{\partial z}-\frac{1}{\rho_t C_t^2}\frac{\partial \sigma_z}{\partial t}+g\frac{D}{2}\frac{\upsilon\rho_f}{Ee}\frac{\partial H}{\partial t}=\frac{\partial I_{\sigma_z}}{\partial t}-\rho_f g\frac{D\upsilon}{2e}\frac{\partial I_{\bar{H}}}{\partial t}$$
$$, C_t^2=\frac{E}{\rho_t} \quad (13)$$

In which C_t is shear wave velocity and ρ_t is density of pipe material.

$$\frac{\partial \dot{u}_z}{\partial t}-\frac{1}{\rho_t}\frac{\partial \sigma_z}{\partial z}=\frac{\rho_f A_f}{\rho_t A_t}\frac{fV|V|}{2D}+gsin\theta \quad (14)$$

A_f and A_t are flow cross section and pipe cross section respectively. For solving Fluid-Structure Interaction, equation 13 and 14 along with fluid continuity and momentum equations, and transfer Matrix are applied for example in a tank-pipe-valve system [8].

5.3 Boundary condition

Boundary conditions have always been the major part of every mathematical model and involve equations in terms of unknown parts that are true only on boundaries. To model the effect of junction interaction, boundary conditions need to be proportional to the numerical method applied in solving equations, and boundary condition again must be applied appropriately in numerical solution process. It is always the most significant part in every solution method [9].

The effect of junction interaction occurs only if some parts of structure, in which momentum shift takes place, are not perfectly connected on the ground. To model the effect of junction interaction, structure dependent hydraulic parameters are used on boundary condition and on structure boundary condition, hydraulic dependent values are used. Therefore, no changes made on fluid structure differential equations, and boundary condition is the only connector between them. Thus we need to prepare boundary condition in another way for structure and hydraulic equations. Each boundary condition used in solving equations, will have a change in their relationship when their nodes are entirely proved structurally. Here we're going to take a look at these changes.

Boundary condition is different in valve, elbow, closed end, pump, and other facilities used in a hydraulic system. Thus, their relationships must be extracted individually. Here, we explain these relationships in valve and tank [9].

1. Tank (with one open end)

If pipe is fixed in this end, we have:

$$D_r=\begin{bmatrix}0&0&1&0\\0&1&0&0\end{bmatrix},\quad f_r(s)=[u_g(s)\quad 0]^T \quad (15)$$

2. Closed valve or closed end with m mass

(a) Fixed pipe

$$D_R=\begin{bmatrix}1&0&0&0\\0&0&1&0\end{bmatrix},\quad f_R(s)=[u_g(s)\quad u_g(s)]^T \quad (16)$$

(b) Free pipe for moving along the axis

$$D_m=\begin{bmatrix}1&0&-1&0\\0&g\rho_f A_f&\pm sm&g\rho_t A_t\end{bmatrix},\quad f_m(s)=[0\quad \pm R_l(s)]^T \quad (17)$$

As A_f and A_t are pipe internal cross section and pipe wall cross section respectively. m is valve mass or closed end. ± is determined according to coordinate system. R_l is the conversion of external stimulus Laplace at the end in question. R_l is explained as below:

$$R_l=A_r\sqrt{E_r\rho_r}(e^{-sTc}-1)/s, \quad (18)$$

6- Solving by transfer matrix

In this method, four first-order partial differential equations are written like the below matrix.

$$A\frac{dy(z,t)}{dt}+B\frac{dy(z,t)}{dt}+cy(z,t)=r(z,t) \quad (19)$$

In which y(z,t) and r(z,t) are vectors of unknown parts and matrixes A,B,C are matrix of fixed coefficients. According to mentioned equations, matrixes and vectors are:

$$y=\begin{bmatrix}V\\H\\u_z\\\sigma_z\end{bmatrix}\quad r=\begin{bmatrix}-\dfrac{fv|v|}{2D}\\ \rho fg(\upsilon^2-1)\dfrac{D}{e}\dfrac{\partial I}{\partial t}\tilde{H}\\ \dfrac{\rho_J A_f}{\rho_t A_t}\dfrac{fv|v|}{2D}+g\sin\theta\\ \dfrac{dI\sigma_t}{dt}-\rho_J g\dfrac{D\upsilon}{2e}\dfrac{dI}{dt}\tilde{H}\end{bmatrix} \quad (20)$$

$$A=\begin{bmatrix}1&0&0&0\\0&g/c_f^2&0&0\\0&0&1&0\\0&\rho_f&0&\dfrac{1}{\rho_t c_f^2}\end{bmatrix},$$

$$B=\begin{bmatrix}0&g&0&0\\1&0&-\lambda D&0\\0&0&0&-\dfrac{1}{\rho_t}\\0&0&1&0\end{bmatrix},\ C_t^2=\frac{E}{\rho_t}, \quad (21)$$

$$C_f^2=\frac{1}{\rho_f}\left[\frac{1}{k}+\frac{2R(1-\upsilon^2)}{eE}\right]^{-1}$$

In equation 21, L stands for pipe ength, υ is Poisson ratio, R shows pipe internal radius, e demonstrates pipe wall thickness, E indicates elastic modulus, g is gravity, and K is fluid bulk modulus. In equation 20, a_{42} and b_{23} are Poisson coupling. By converting Laplace on equation, we have:

$$SA(s)Y(z,s)+\beta\frac{dy(z,s)}{d\tau}=\bar{r}(z,s) \quad (22)$$

We have:

$$Y(z,s)=L(y(z,t)) \quad A(S)=A \quad (23\_1)$$

$$\bar{r}(z,s)=L(r(z,t)+A(s)y(z,0) \quad (23\_2)$$

In which y(z,0) vector is initial condition. Eigenvalues of expressions like this can be computed through$|B-\lambda A|=0$ like below.

$$C_f=\lambda_1=-\lambda_2=\sqrt{\frac{1}{2}\left\{q^2-\sqrt{q^4-4c_f^2c_t^2}\right\}},$$

$$C_t=\lambda_3=-\lambda_4=\sqrt{\frac{1}{2}\left\{q^2+\sqrt{q^4-4c_f^2c_t^2}\right\}},$$

$$q=C_f^2+C_t^2+2\upsilon^2\frac{\rho_f}{\rho_t}\frac{R}{e}C_f^2 \quad (24)$$

Matrix Λ, a diagonal matrix with achieved eigenvalues is as below:

$$\Lambda=\operatorname{diag}\{\lambda_1(s),-\lambda_1(s),\lambda_3(s),-\lambda_3(s)\} \quad (25)$$

One solution is:

$$\tilde{\phi}(\tau,s)=S(s)\tilde{\eta}(\tau,t) \quad (26)$$

Computing S and η:

$\Lambda(S)=S^{-1}\ A^{*-1}\ B\ S(s)$

$S(s)=(\xi_1(s)\ \xi_2(s)\ \xi_3(s),\ \ldots)$ (27)

Each $\xi_i(s)$ belong to λ_i

$$\tilde{\eta}_i(z,s)=\tilde{\eta}_{0i}(s)e^{-sz/\lambda_{i(s)}}+\tilde{\eta}_{ri}^{*}(z,s)$$
$$i=1,2,\ldots,N \quad (28)$$

n which $\tilde{\eta}_{ri}$ can be computed by using boundary condition. And $\tilde{\eta}_{ri}^{*}$ shows the private answer.

$$\tilde{\eta}_{ri}^{*}(z,s)=\frac{se^{-sz/\lambda_{i(s)}}}{\lambda_{i(s)}}\int^{z}\tilde{\eta}_{ri}(z^*,s)e^{sz^*/\lambda_{i(s)}}dz^*$$
$$i=1,2,\ldots.N \quad (29)$$

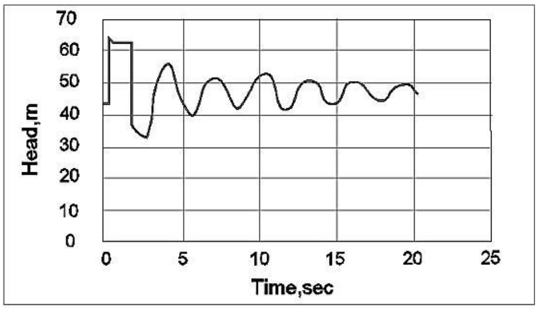

Figure 2: Pressure changes as compared to time according to research conducted by covas et al [10].

Don't forget that if $\tilde{\eta}_{ri}^{*}$ is independent of z, $\tilde{\eta}_{ri}^{*}$ (r,z)equals $\tilde{\eta}_{ri}$ (s).

The vector of η̃ is written like this:

$$\tilde{\eta}(z.s)=\mathrm{E}(z,s)\tilde{\eta}_0(s)+\tilde{\lambda}_r^{*}(z,s) \quad (30)$$

In which diagonal matrix E is as below.

$$E(z,s)=diag\left(e^{-sz/\lambda_1(s)},e^{-sz/\lambda_2(s)},e^{-sz/\lambda_3(s)},e^{-sz/\lambda_4(s)}\right) \quad (31)$$

Table 1: Input data

Fluid special mass	Stress wave velocity	Pressure wave velocity	head	Flow rate	Bulk modulus
$1000kg/m^3$	630m/s	395 m/s	45m	1.o11/s	2.2 GPa

Poisson coefficient	Yung Modulus	Wall thickness	Internal diagonal	length
0.46	1.43GPa	6.3 mm	50.6mm	277m

Table 2: results of creep functions in

$Q = 1.01\frac{l}{s}$, $C_f = 395\frac{m}{s}$

Kelvin-Woigt	τk,Jk (10^-10 Pa-1)				
5	τ1=0.05s	0.5	1.5	5	10
	J1=0.057	1.054	0.9051	0.2617	0.7456

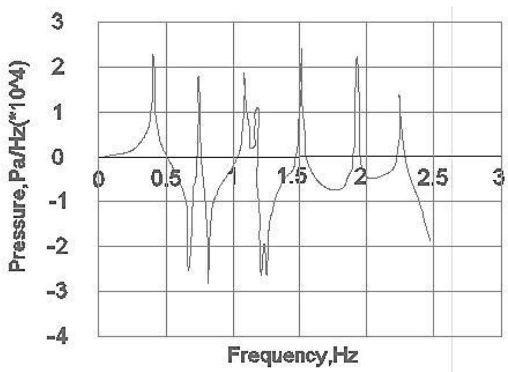

Figure 3: Pressure changes as compared to frequency

7- Flow in viscoelastic pipes

Water hammer test is conducted in a polyethylene pipe system. This test was performed by covas et al [7] and is also known as Imperial College [10, 11]. This test can be implemented like a tank-pipe-valve system. Input data is available in tables 1 and 2.

8- Conclusions

In sum, the major findings achieved after studying the effect of viscoelastic pipe networks in frequency domain are as the follow:

The capability and applicability of the transfer matrix method, which was the main objective of the study, were proved by using resource models.

Investigation of interaction in frequency domain indicated that boundary conditions must be restricted as much as possible.

Single pipes and less complicated systems in frequency domain can be better analyzed.

Fluid pressure was studied when system was under vibration effect and it was observed that under the influence of structure – interaction, system natural frequencies were intensified. It indicates the destructive effect of viscoelasticity and FSI effect.

In comparison with other methods (like FEM), TMM has another advantage that the length of used matrixes rarely increase increased by modeled system size.

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