Power Losses in the Engine-Transmission Installation of the Mobile Unit under Probabilistic Load Conditions

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Abstract
The article reviewed a method for assessing the influence of a probabilistic load on the amount of energy loss in an engine-and-transmission unit (ETU) of a mobile aggregate. The method is based on the usage of a random processes theory. The random process is a sum of harmonic components with a random initial phase and constant amplitude. The image of the random process corresponds to an arcsine distribution, in which the normal distribution law is approximated by an arcsine function. A peculiarity is that the approach allows you to obtain characteristics that reflect the dependence of the energy performance of the engine-and-transmission installation on the frequency of oscillations of the rotational speed of the crankshaft of an internal combustion engine. The frequency characteristics of the mathematical expectation of the effective power of the internal combustion engine, dissipative losses in the dynamic systems of the internal combustion engine and transmission are obtained as a result of all calculations. We propose a generalized criterion to assess the energy efficiency of a mobile unit under the influence of a probabilistic load.

Keywords: engine-and-transmission unit (ETU), random processes theory, internal combustion engine, dynamic systems, mathematical expectation

1 Introduction
Over recent years through studying the energy indicators of machine-tractor aggregates, the probability-statistical characteristics of random processes interacting with both power transmission and the engine are widely used (1, 2, 3, 4). The approach allows us to evaluate the energy performance of the machine-tractor aggregate (MTA) in conditions close to real. Common to all tractor vibrations is the energy of only one source is expended on their excitation and maintenance - the engine. So, the level and intensity of all vibrations accompanying the operation of the tractor affect its energy performance, and, consequently, productivity and fuel savings.

Based on (В.Н. Болтинский) V.N. Boltsinskii’s theoretical and experimental studies, engine power and economic indicators, due to load fluctuations, decrease, compared to the indicators when loading with a constant torque (5). He introduced the concept of engine power utilization coefficient, which is equal to the ratio the maximum power received during the operation of the engine with a variable load, to the maximum power received during standard brake tests. It has been established that the main reason for reducing the energy performance of the engine is the fluctuation of the angular velocity of the crankshaft, and these fluctuations, in turn, are determined by the reduced moment of inertia, the degree of unevenness of the moment of resistance, the degree of insensitivity of the regulator, and the adaptability coefficient of the IC engine.

Following studies of numerous authors were aimed at a deeper study of the processes occurring in the IC engine under the unsteady nature of the load (6, 7, 8, 9, 10, 11), to search for ways to improve engine efficiency (l, 2, 7, 12, 13, 15, 16, 17), on the methods development for assessing and forecasting the main technical and economic indicators of IC engine (l, 11, 13, 18, 19, 29). In most works it is noted that one of the main reasons for the decline in engine performance is the non-linearity of its regulatory characteristic (l, 6, 12, 21, 22), and in this regard, negative phenomena are observed more significantly, the closer the IC engine load to the nominal, the less adaptability coefficient, the greater the amplitude of the oscillations, the smaller the reduced moment of inertia. Due to the fact that the efficiency of the IC engine...
under the unsteady nature of the load is largely determined by its dynamic qualities, scientific research aimed at creating mathematical models has been developed (11, 23, 24, 25). With their help, transient processes are studied during faults and load shedding, and, based on frequency methods, technical and economic indicators of IC engine in dynamic modes (4, 11).

Mathematical method in the form of linear and nonlinear differential equations is widely used in these works. Some authors focus on the frequency method using statistical models of random processes as an input signal (4, 18, 23).

The unsteady nature of the load influences the energy processes in the engine-transmission installation as a whole. Due to fluctuations in the rotational speed of the shafts, additional power losses due to friction and slipping in kinematic pairs are formed. The nature of the losses has received a fairly complete interpretation in the works (20, 26).

The next group of theoretical methods is associated with the study of mathematical models composed of equations describing equivalent circuits of dynamic systems. The first theoretical works devoted to the study of the dynamic properties of mechanical systems, based on the principles and principles adopted in electrical engineering, appeared in the 30s (27, 28, 29, 30, 31). These studies substantiate the possibility of using electromechanical analogies when considering issues related to the study of oscillatory processes in mechanical systems. This approach made it possible to transfer to the ready-made mechanical system a number of conclusions obtained in some well-developed branches of electrical engineering (for example, in the theory of four-terminal networks, in the theory of electric filters, etc.). In the papers under consideration, the concept of equivalent circuits is given and methods for constructing them are shown. The equivalence of two systems - electrical and mechanical - is justified by the identity of the Lagrange equations presented in generalized coordinates. From them equations are derived that describe mechanical systems and are similar to Kirchhoff's equations. So, for example, an electric circuit composed of inductance L, capacitance C and resistance R is described by the equation of the second Kirchhoff law. Similarly, a mechanical system is described by equations obtained on the basis of the de Alambère principle. These works give an idea of the fundamental possibilities of an approach based on the formation and study of equivalent circuits of mechanical systems.

The analysis of technological processes carried out both on industrial and agricultural machine-tractor aggregates suggests that the maximum spectral density of load fluctuations falls precisely on the low-frequency region.

In (32), there is a classification of processes occurring in a dynamic system. High-frequency vibrations seem to be associated with individual mechanical links, which constitute a separate type element through hard links. It is argued that low-frequency oscillations occur in a system of typical elements and cause fluctuations in the speed of rotation of the shafts. A common drawback of all the theoretical works discussed above is that they do not reveal at all or weakly reveal the questions of studying the dynamic properties of systems composed of components of a different physical nature (electrical, hydraulic, mechanical).

In general, from a review of analytical methods for studying the dynamic properties of power transmission, it can be concluded that the scope of existing methods is limited to one specific physical nature (mechanical, hydraulic, electrical). It can be noted as a drawback that they do not consider complex dynamic systems consisting of a set of heterogeneous elements.

The features of power transmissions of various physical nature and the related features of the methods do not allow the use of the latter as universal ones in substantiating the rational type of continuously variable transmission (33-37).

The analysis of trends in modern tractor engineering allows us to conclude that one of the main goals of improving tractors as the main energy tools for mobile crop production is to increase their productivity. Therefore, the industry has developed a direction of research work related to the search for ways to more fully load the diesel engine, to improve the dynamics of the tractor in operating conditions and, in general, to increase the fuel economy and productivity of the MTA. Currently, the following paths have been identified: the introduction of progressive power transmissions, the correction of the static and dynamic characteristics of internal combustion engines. It implies stepless hydrodynamic, hydrostatic, electric transmissions and an increase, within the limits of the steepness of the corrector, of the speed characteristic of the internal combustion engine.

According to experts, the situation in the field of improving the manufactured equipment and developing promising modifications of tractors and agricultural machines can be seriously improved by providing designers and researchers with universal engineering methods for calculating the energy and dynamic indicators of MTA, which are invariant to the physical heterogeneity of the mechanisms of the unit, allowing for their static and dynamic characteristics, speed and load conditions of the machine-tractor unit and ultimately, giving the opportunity at the design stage of new technology to obtain reliable comprehensive information about the MTA indicators of interest in the probabilistic nature of the external load.

Let us give a physical interpretation to the procedure for calculating the energy indicators of a machine-tractor unit using the spectral density of a random process. As is known, spectral density is a function that characterizes the frequency distribution of elementary dispersions (38). From here, in accordance with (39), one can proceed to the frequency distribution of the amplitudes of harmonic processes:

$$AA(\omega) = \sqrt{2\Delta A(\omega)}.$$  \hspace{1cm} (1)

So, the input variables in a random process obeying the arcsine distribution can be represented by a set of amplitudes of harmonic oscillations distributed over a frequency (Figure 1), and then it is legitimate to talk about the frequency characteristics of the mathematical expectation of the effective power of the internal combustion engine and other indicators. $M_t$ - dependence of the torque on the shaft of an internal combustion engine (ICE) on the speed of rotation of the shaft; $N_t$ is the dependence of the effective power on the
ICE shaft on the shaft rotation speed; \( M(M_k) \) and \( M(N_e) \) are the dependences of the mathematical expectations of the torque and the effective power of the internal combustion engine on the shaft rotation speed; \( \phi(n) \) - normal and arcsine distribution laws of a random process and harmonic components with a random initial phase and constant amplitude; \( n(t) \) is a random process of the speed of rotation of the ICE shaft; \( n(t)_k \) are the harmonic components of the random process of the speed of rotation of the ICE shaft.

This distinguishes the technique considered in the work from that described in (1).

### 2 Research Methods

We consider the non-normalized spectral density of the ICE crankshaft rotational speed as the frequency distribution of harmonics dispersions (38), use the provisions of (1) to determine the amplitudes of harmonic vibrations for different frequencies

\[
\Delta \Omega(\omega) = \sqrt{2S(\omega)_{DCS}}. \tag{2}
\]

Substituting (2) into the expression for calculating the expectation value of the effective ICE power under the assumption of the arcsine distribution of a random function of rotation speed (1), we get

\[
[M(N_e)](\omega) = 0.5[a'\Omega_k^2 + b^2 + 0.5b^2(\Delta \Omega)^2(\omega)]\pi^{-1} \times
\]

\[
\times \left[ a_0^2 + b^2 + 0.5b(\Delta \Omega)^2(\omega) \right]^{\Delta \Omega(\omega)} +
\]

\[
+ \frac{1}{2\pi} (3\Omega_K^2 - \Omega_1^2)\sqrt{\Delta \Omega^2(\omega) - (\Omega_1 - \Omega_K)^2}.
\]

Let’s look onto the methodology for determining, based on the obtained frequency response, the integral value of the mathematical expectation of the effective power of the internal combustion engine.

1. The relation is determined to determine the frequency response of the expectation value of power loss

\[
[M(\Delta N_e)](\omega) = N_e - [M(N_e)](\omega). \tag{4}
\]

2. By integrating the frequency response (4), we get the mathematical expectation of the total power loss

\[
M(\Delta N_e) = \sum_{K=1}^{n} [M(\Delta N_e)](\omega_k) \Delta \omega_k, \tag{5}
\]

\( K \) is a harmonic order.

3. The expectation value of the effective power of the internal combustion engine is calculated, corresponding to the influence of the spectrum of speed fluctuations

\[
M(N_e) = N_e - M(\Delta N_e). \tag{6}
\]

Numerous experimental and theoretical studies (40, 41, 42) suggest that vibrational processes cause additional dissipative losses in a dynamic system. From the theory of oscillations (41, 42) it is known that dissipation losses due to the action of harmonic oscillations on a system are determined from the relation

\[
\Delta N = \Omega M \cos \varphi. \tag{7}
\]

\( \Omega \) is rms-speed for one period of oscillation,

\( M \) is a same torque.

If \( \omega_0 \) is an instantaneous state of velocity, the root-mean-square will be:
\[ \Omega = \sqrt{\frac{1}{T} \int_0^T \omega_0^2 \, dt} = \sqrt{\frac{\Delta \omega^2}{T} \int_0^T \sin^2(\omega_0 t) \, dt} = \frac{\Delta \omega}{\sqrt{2}}. \]  

Similar to torque
\[ M = \sqrt{\frac{1}{T} \int_0^T m^2 \, dt} = \left(\frac{\Delta M^2}{T} \int_0^T \sin^2(\omega_0 t) \, dt\right)^{\frac{1}{2}} = \frac{\Delta M}{\sqrt{2}}. \]  

\( T \) is the period of harmonic oscillation, \( \Delta \omega, \Delta M \) are the amplitudes of the oscillations of speed and torque, \( m \) is the instantaneous value of the torque, \( \omega_0 \) is the angular frequency of oscillations.

Theoretical studies performed by a number of authors suggest that the above reasoning can be applied to random functions with sufficient accuracy. In this case, the standard deviations can be obtained from the relations
\[ \sigma_M = \sqrt{D(M), \sigma_\Omega = \sqrt{D(\Omega)}} \]  

and to determine the amplitudes of the oscillations acceptable ratio
\[ \Delta M = \sqrt{2} \sigma_M, \quad \Delta \Omega = \sqrt{2} \sigma_\Omega. \]  

Under the probabilistic nature of the external load, power losses in the dissipative elements of the engine-transmission installation of the mobile unit can be found by the expression
\[ \Delta N(\omega)_{ETI} = \sqrt{\frac{D(M)}{D(\Omega)}} \sigma(\omega)_{mle} \sigma(\omega)_{fle} \cos(\omega). \]  

\( \sigma(\omega)_{mle}, \sigma(\omega)_{fle} \) are normalized spectral densities of processes on the driving-wheel (leading wheel) of the tractor \( \varphi(\omega) \) - phase angle between torque and speed at a fixed harmonic.

Let’s find an expression for calculating the phase shift at a fixed harmonic. We will use the equivalent circuit of the engine-transmission installation (Fig.2). Full mechanical conductivity of the ETI dynamic system.

\[ Y_0 = Y_23 + j\omega e_23 + \frac{Y_1 + j\omega e_1 [\cos\varphi(\omega) + j\sin\varphi(\omega)]}{Y_1 + j\omega e_1 [\cos\varphi(\omega) + j\sin\varphi(\omega)](\theta_1 + j\omega \theta_2) + 1}, \]  

\[ Y_{etl} = Y_4 + Y_{et} + \frac{Y_0}{Y_0(\theta_4 + j\omega \theta_5) + 1}. \]  

Phase angle
\[ \theta_{etl} = \text{arctg} \frac{\text{Im} Y_{etl}}{\text{Re} Y_{etl}}. \]  

\( e_1 \) is the elasticity compliance in the dynamic system of a heat engine; \( Y_1 \) - mechanical conductivity, characterizes the speed loss in the same place; \( Y_{el2} \) - mechanical conductivity, the reciprocal of the mechanical resistance, characterizes the loss of torque in the same place; \( J_1 \) is the moment of inertia of the engine flywheel and the translationally and rotationally moving masses rigidly connected with it; \( Y_{el3} \) - mechanical conductivity, characterizes the speed loss in the dynamic systems of the first and second converters; \( e_{el2} \) - elastic coupling compliance in dynamic systems of the first and second transducers; \( Y_{el6} \) - mechanical conductivity, the reciprocal of the mechanical resistance, characterizes the loss of torque to the dynamic system of the second converter; \( J_2 \) is the moment of inertia of the translationally and rotationally moving masses rigidly connected with the second transducer; \( Y_{el5} \) - mechanical conductivity of the engine slipping section; \( \lambda_d \) - a parameter reflecting the resistance of an external load.

In order to establish how losses are distributed between the internal combustion engine and the transmission, we find the corresponding coefficient and call it the coefficient of the level of losses in the tractor transmission. We will again use the equivalent circuit. It is convenient here to turn to some provisions of the theory of electric circuits (42). In accordance with them, the ratio of power on any part of the circuit to the total power supplied to the input of the dynamic system is equal to the ratio of the resistances (conductivities) of the corresponding sections.

For our case, we can write:
\[ \frac{\Delta N(\omega)_{el}}{\Delta N(\omega)_{etl}} = \frac{\text{Re} Y_{el}}{\text{Re} Y_{etl}} = \lambda(\omega), \]  

\( Y_1 \) - mechanical conductivity of a dynamic ICE system,
\( Y_{el} \) - mechanical conductivity of the MTA dynamic system, found when calculating the angle of phase shift from (14)
\[ Y_e = \frac{Y_1 + j\omega e_1 [\cos\varphi(\omega) + j\sin\varphi(\omega)]}{Y_1 + j\omega e_1 [\cos\varphi(\omega) + j\sin\varphi(\omega)](\theta_1 + j\omega \theta_2) + 1}. \]  

If we know \( \lambda(\omega) \), the components of losses in the internal combustion engine and transmission are calculated. So the losses in the engine are determined by the ratio
\[ \Delta N(\omega)_{el} = \Delta N(\omega)_{etl} \cdot \lambda(\omega), \]  

transmission losses
\[ \Delta N(\omega)_{tr} = \Delta N(\omega)_{etl} \left[ 1 - \lambda(\omega) \right]. \]  

We express the components of the power loss in the ETI in relative terms. The relative amount of losses in the ICE
\[ \lambda_1(\omega) = 1 - \frac{\Delta N(\omega)_{el}}{Y_e}, \]  

And in transmission
\[ \lambda_2(\omega) = 1 - \frac{\Delta N(\omega)_{tr}}{Y_e}. \]
Frequency response of the ICE effective power factor

\[ \lambda(\omega)_{Ne} = \frac{[M(N_e)](\omega)}{N_e} . \]  

The convenience of the obtained coefficients lies in the fact that with their help it is possible to compare the levels of all loss components and formulate a generalized criterion that allows us to evaluate the effectiveness of the engine-transmission installation as a whole

\[ k_{\text{crit}}(\omega) = \lambda_1(\omega) \lambda_2(\omega) \lambda(\omega)_{Ne} \to \max. \]  

Example of calculating energy losses of tractor engine-transmission installation

\[ S(\omega) \] is the abnormal spectral density of the rotational speed of the crankshaft of an internal combustion engine (ICE); \( M(N_e) \) is the mathematical expectation of the effective power of the internal combustion engine;

\( A(\omega) \) is an amplitude-frequency characteristic of the transfer function of the tractor engine-transmission installation; \( \Delta N_e \) is energy loss due to the variable component of the load in the transmission; \( \Delta N_t \) energy losses due to the variable component of the load in the engine; \( \sigma(\omega) \) is the normalized spectral density of the rotational speed of the crankshaft of an internal combustion engine.

To estimate the losses caused by the impact on the dynamic system of a set of harmonics, we integrate relations (3.64) and (3.65):

\[ \Delta N_e = \sum_{k=1}^{n} \Delta N(\omega_k)_{Ne} \Delta \omega , \]
\[ \Delta N_t = \sum_{k=1}^{n} \Delta N(\omega_k)_{Tr} \Delta \omega . \]

\( k \) is a harmonic order.

3 Conclusions

1. A random process following the normal distribution law can be described by an arcsine distribution.
2. Input variables in a random process following the normal distribution law can be represented by a set of harmonic oscillations with a random initial phase and constant amplitude, i.e., following the arcsine distribution.
3. Under the probabilistic nature of the external load, the mathematical expectation of the effective ICE power and power loss in the dissipative elements of the engine-transmission installation of the mobile unit can be reflected in the frequency characteristics.
4. It is possible to establish the frequency range of load oscillations on the driving wheels of the mobile unit based on the analysis of the data of frequency characteristics. It significantly affects the effective power of the ICE and the amount of dissipation loss in dynamic systems of the engine and transmission.
5. Modeling the dynamic system of the engine-transmission installation of a mobile unit can be carried out on the basis of equivalent circuits, allowing calculations using methods that have been widely used in the theory of electrical engineering.

References